

Luttinger sum rule for finite systems of correlated electrons

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(Dated: February 6, 2008)

The validity of the Luttinger sum rule is considered for finite systems of interacting electrons, where the Fermi volume is determined by location of zeroes of Green's function. It is shown that the sum rule in the paramagnetic state is evidently violated within the planar t - J model at low doping while for the related Hubbard model, even in the presence of next-nearest-neighbor hopping, no clearcut exception is found.

PACS numbers: 71.27.+a, 71.18.+y, 71.10.-w

I. INTRODUCTION

In last decades extensive experimental and theoretical investigations of high-temperature superconductors and other novel materials with strongly correlated electrons raised several fundamental questions regarding the Fermi liquid (FL) concept. One of the central assumptions in the FL theory is the existence of well defined Fermi surface with the volume, independent of electron-electron interactions. The latter property has firm basis in the Luttinger sum rule (LSR)^{1,2,3}, which represents the relation between the electron density and the poles or zeroes of the single-particle Green's function (GF) $G(\mathbf{k}, \omega)$ at the chemical potential.

To settle this issue several authors reconsidered the LSR, its validity and extensions regarding different aspects in connection with strongly interacting electrons. It has been shown that in one dimension (1D) LSR remains robust, although the usual FL is replaced by the Luttinger-liquid phenomenology⁴. LSR is valid even for models of strongly correlated electrons as the Kondo lattice model⁵. Recently, the LSR in the Mott insulating state has attracted attention. In this case the chemical potential is in the gap so there is no Fermi surface, however the LSR can still apply for zeroes of GF and in this way the definition of Fermi surface is generalized to the concept of 'Luttinger' surface⁶ and corresponding Luttinger volume (LV). In the insulator the LSR should remain valid for models with the electron-hole symmetry as the Hubbard model with the nearest-neighbor hopping on a bipartite lattice⁷. On the other hand, extensions of the latter model can lead to the violation of the LSR in the insulating state⁸, or a novel form in ladder systems⁹. The LSR has to be also reformulated in the case of emergence of long range order¹⁰. Clearly, the most challenging question is the metallic state of strongly correlated electrons in the absence of any broken symmetry. There are indications from photoemission experiments that at least on some hole-doped cuprates the Luttinger volume is not conserved^{11,12}. Similar conclusions can be drawn from numerical studies of related relevant models as the t - J model¹³ and Hubbard model¹⁴ near the Mott insulating state although deviations are modest.

In this paper we consider the LSR on finite systems of correlated electrons. We use the observation that the main ingredients of the theorem¹ are valid for finite systems, exploited rarely so far¹⁵. The underlying idea is that in this way one can

directly test nontrivial models in different parameter regimes on the validity of LSR and the topology of the related Fermi surface and LV. While in this way it is still hard to prove the Luttinger theorem in the most relevant thermodynamic limit, it is much easier to show on its breakdown. As shown later, one should be careful in the classification of scenarios of LSR violation, since only some of them could remain relevant up to the thermodynamic limit.

We investigate as examples two prototype models of strongly correlated electrons, most frequently studied in connection with superconducting cuprates, namely the two-dimensional (2D) Hubbard model and the t - J model on a square lattice. For comparison, we perform tests also for corresponding 1D models whereby the 1D Hubbard model is exactly solvable^{16,17} and the LSR should remain valid in the thermodynamic limit. Within our numerical limitations presented results show that also 2D Hubbard model generally fulfills the LSR in the whole parameter range of the paramagnetic state, even in the presence of the next-nearest-neighbor hopping t' . Still, we find some exception which we classify as not clearcut. On the other hand, the 2D (and even 1D) t - J model reveals evident deviations from LSR close to half filling.

The paper is organized as follows. In Sec. II we briefly summarize the formalism underlying the Luttinger sum rule, in particular its relevance and application for finite-size systems. We introduce the tight-binding models for interacting electrons, whereby the Hubbard model is the simplest one allowing the study of continuous development from the non-interacting case satisfying trivially LSR into the strong-correlation regime. The approach of avoiding the degeneracy with twisted boundary condition is described. We also classify several possible scenarios of the LSR breakdown in finite systems. In Sec. III we present numerical tests of the LSR for the Hubbard model and the t - J model, both on 1D chains and 2D square lattice. In particular, we try to pinpoint clearcut cases of the LSR breakdown, which are evident for 2D t - J model near half-filling. Sec. IV is devoted to conclusions and discussion.

II. LUTTINGER SUM RULE

A. Formalism for finite systems

We consider here the homogeneous system of interacting electrons on a lattice with periodic boundary conditions. In this case the LSR^{1,2,3} can be expressed as

$$N = \sum_{\mathbf{k}s, G_s(\mathbf{k}, 0) > 0} 1, \quad (1)$$

relating the number of fermions N to the value of zero-temperature $T = 0$ single-particle GF $G_s(\mathbf{k}, \omega = 0)$ at the chemical potential μ . General Tr^1 is in homogeneous case replaced by the sum over \mathbf{k} and spin s .

Let us recall few essential steps in the derivation of Eq. (1)^{1,3} in order to see possible limitations and its proper application for finite systems. Generally one can express

$$N = \frac{1}{\beta} \sum_l \sum_{\mathbf{k}s} \mathcal{G}_s(\mathbf{k}, \omega_l) e^{i\omega_l 0^+}, \quad (2)$$

where $\mathcal{G}_s(\mathbf{k}, \omega_l)$ is the $T > 0$ propagator at Matsubara $\omega_l = 2\pi(2l + 1)/\beta$ and $\beta = 1/T$ (we use units $\hbar = k_B = 1$). With the definition of the self energy $\Sigma_s(\mathbf{k}, \zeta) = \zeta - \epsilon_s(\mathbf{k}) - 1/\mathcal{G}_s(\mathbf{k}, \zeta)$ Eq. (2) can be rewritten as $N = I_1 + I_2$ ^{1,3},

$$I_1 = -\frac{1}{2\pi i} \sum_{\mathbf{k}s} \int_{\Gamma} \frac{\partial}{\partial \zeta} \ln(\mathcal{G}_s(\mathbf{k}, \zeta)) e^{\zeta 0^+} \frac{1}{e^{\beta \hbar \zeta} + 1} d\zeta \quad (3)$$

$$I_2 = \frac{1}{2\pi i} \sum_{\mathbf{k}s} \int_{\Gamma} \mathcal{G}_s(\mathbf{k}, \zeta) \frac{\partial}{\partial \zeta} \Sigma_s(\mathbf{k}, \zeta) e^{\zeta 0^+} \frac{1}{e^{\beta \hbar \zeta} + 1} d\zeta,$$

where integration path Γ clockwise encloses real axis. The central point of the proof of the LSR is the observation that I_2 in Eq. (4) vanishes after limiting $T \rightarrow 0$. The latter is argued with the construction of the functional Y' , which is the contribution of all closed linked skeleton diagrams to thermodynamic potential. I_2 can be represented as a full derivative of Y' , hence $I_2 = \partial Y' / \partial \delta \epsilon = 0$ ^{1,3}. We note that the existence and convergence of Y' is shown within the perturbation theory, which applies also for finite systems. It should be pointed out that deviations from LSR in concrete case discussed later on can be traced back to the nonvanishing I_2 and in this way to the breakdown of perturbation theory for Y' .

In regular cases we have $N = I_1$ which for $T \rightarrow 0$ reduces to

$$N = -\frac{1}{2\pi i} \sum_{\mathbf{k}s} 2\text{Im} \left[\int_{-\infty}^0 d\zeta \left\{ \frac{\partial}{\partial \zeta} \ln(G_s(\mathbf{k}, \zeta)) \right\} \right]. \quad (4)$$

In finite systems we can express the GF for N electrons explicitly in terms of eigenstates of systems with $N-1, N, N+1$ particles,

$$\begin{aligned} G_s(\mathbf{k}, \zeta) &= \sum_m \frac{|\langle m_{N-1} | c_{\mathbf{k}s} | 0_N \rangle|^2}{\zeta + \mu_N - (E_0^N - E_m^{N-1})} + \\ &+ \sum_l \frac{|\langle l_{N+1} | c_{\mathbf{k}s}^\dagger | 0_N \rangle|^2}{\zeta + \mu_N - (E_l^{N+1} - E_0^N)}. \end{aligned} \quad (5)$$

μ_N in Eq. (5) is according to derivation¹ defined by the grand-canonical relation $N = T \partial(\ln \Omega) / \partial \mu$ in the limit $T \rightarrow 0$. The latter gives uniquely $\mu_N = (E_0^{N+1} - E_0^{N-1})/2$ under the provision that E_0^N is a concave function of N or at least $E_0^{N+1} + E_0^{N-1} - 2E_0^N > 0$, which is equivalent to the phase stability of the ground state (g.s.). It should be noted that in finite systems the latter condition is not fulfilled in certain case which can be signature of a physical instability but as well just a finite-size effect. Another source of ambiguities in finite size systems can arise from the degeneracy of the ground state $|0_N\rangle$, which can be partly removed by introduction of appropriate fields and twisted boundary conditions.

It is easy to see that the contributions to Eq. (4) come from poles and zeros of $G_s(\mathbf{k}, \omega < 0)$. The latter appear in pairs, and give residuums ± 1 , respectively, mostly cancelling each other. As a rule we establish that finally the LSR, Eq. (1), is determined by the unpaired zero of GF at $\omega \sim 0$ which can appear either for $\omega \geq 0$ or $\omega \leq 0$. This is a distinction to a macroscopic systems where the Fermi surface is located by the poles of GF at $\omega = 0$. On the other hand, in a finite system the poles of GF are at $\omega \neq 0$, since in Eq. (5) in general $E_0^N - E_0^{N-1} < \mu_N$ and $E_0^{N+1} - E_0^N > \mu_N$ due the choice of μ_N and concavity of E_0^N . Clearly, possible exceptions are when the latter condition is not satisfied as well when the g.s. is degenerate.

B. Tight binding models

In the following we test the validity of LSR for single-band tight-binding models for correlated electrons,

$$H = - \sum_{i,j,s} t_{ij} c_{js}^\dagger c_{is} + H_{int}, \quad (6)$$

where the prototype model is the Hubbard model with local repulsion

$$H_{int} = U \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (7)$$

and the second model is the t - J model discussed lateron. We consider finite systems with N_0 sites on 1D chain and 2D square lattice. Besides the nearest-neighbor hopping $t_{ij} = t$ we investigate also the next-nearest-neighbor hopping $t_{ij} = t'$, since the latter breaks the electron-hole symmetry, e.g. in the Hubbard model, which may affect the LSR⁷. The Hubbard model has the advantage that by increasing U one can study continuous development from the reference system of noninteracting electrons (NIE) on a lattice, where the LSR is trivially satisfied, into a strong correlation regime where the breakdown of LSR can appear in several ways. It is evident that the interaction U can be treated within the standard perturbation theory and therefore the arguments underlying the proof of LSR¹ should apply.

We note that in finite systems even for $U = 0$ a degeneracy of the g.s. can appear in the case of periodic boundary conditions due to discrete single-particle $\mathbf{k} = \mathbf{k}_i, i = 1, N_0$. This

can be removed by introducing the twisted boundary conditions which are achieved by replacing $t_{ij} \rightarrow t_{ij} \exp(i\theta \mathbf{r}_{ij})$ while the interaction in Eq. (6) is supposed not to depend on θ . In this case, we are dealing on 2D square lattice with the reference system of NIE with the single-particle dispersion

$$\epsilon(\mathbf{k}) = -2t(\cos \tilde{k}_x + \cos \tilde{k}_y) - 4t' \cos \tilde{k}_x \cos \tilde{k}_y, \quad (8)$$

where $\tilde{\mathbf{k}} = \mathbf{k} + \theta$. To remove degeneracies, it is enough to take $\theta = (\theta_x, \theta_y)$ infinitesimally small, although also finite θ may have a meaning, e.g. by minimizing the g.s. energy $E_0^N(\theta)$.

C. Scenarios of sum-rule violation

Before presenting results of our analysis, let us classify possible ways of the LSR violation. We note that generally eigenstates of Eq.(6) can be sorted with respect to several quantum numbers: number of electrons N , total spin projection S^z (as well as S), total momentum \mathbf{K} , etc. According to the latter several scenarios are possible, e.g., for the Hubbard model, bearing in mind that the reference NIE system is paramagnetic:

I) Turning on $U > 0$ the character of the g.s. $|0_N\rangle$ can change due to the crossing of levels with different symmetries. This is manifested by an abrupt jump of $G(\mathbf{k}, 0)$ and possible violation of LSR. Such a case can be a signature of a macroscopic phase change, e.g., for $S \gg 0^{10}$ indicating a ferromagnetic instability (e.g., for $U \rightarrow \infty$ and $N = N_0 \pm 1$ well known Nagaoka instability), or merely a finite size effect (mostly change of \mathbf{K}).

II) Similar, but more subtle, could be the effect of the level crossing in $|0_{N+1}\rangle$ or $|0_{N-1}\rangle$. In this case, μ_N is determined by the g.s. with $N+1, N-1$ electrons, while $G(\mathbf{k}, 0)$, Eq. (5) does not necessarily contain matrix elements between $N-1, N, N+1$ g.s., if e.g. $|S_{N+1} - S_N| > 1/2$. As in scenario I such a level crossing could be a sign of a macroscopic instability.

III) The g.s. of NIE can be degenerate on finite lattices. The latter can be eliminated by introducing small θ , but is removed also by $U > 0$. Such a case with possible level crossing between different \mathbf{K} at small $U^*(\theta)$ could lead to ambiguity, but is not the problem if the breakdown of LSR appears at larger $U_c \gg U^*$.

IV) To avoid degeneracies of NIE, most rewarding are configurations with closed shells of electrons where we can fix $\theta = 0$. Then most clearcut (excluding scenarios I, II) breakdown of the LSR, Eq. (1), can appear when by increasing $U > U_c$ one or several zeroes of $G(\mathbf{k}, \omega_0)$ cross the chemical potential, i.e. ω_0 changes sign.

III. NUMERICAL RESULTS

In the following we present numerical results obtained for the 1D and 2D Hubbard model and t - J model. In 2D lattices are chosen of the Pythagorean form $N_0 = L^2 + M^2$. To avoid complications in the interpretation we consider only

cases with even N, N_0 . For smaller sizes, $N_0 \leq 8$ one can evaluate $G_s(\mathbf{k}, \omega)$ via Eq. (5) finding all eigenstates using the exact (full) diagonalization within the basis states for given N, S^z . To reach larger sizes, i.e. $N_0 = 16$ for Hubbard model and $N_0 = 20$ for the t - J model, respectively, we calculate the g.s. performing the exact diagonalization with the Lanczos technique, which is then also applied to calculate $T = 0$ $G_s(\mathbf{k}, \omega)$ in a usual way¹⁸. It can be shown that such a procedure yields acute values for quantities of interest, in particular E_0^N, μ_N and $G_s(\mathbf{k}, 0)$.

A. Hubbard model

1D systems: For the start we test the basic $t' = 0$ Hubbard model, Eq. (6), on a chain with up to $N_0 = 14$ sites. As expected we find no violation of the LSR in cases with even N in the investigated range of $0 < U < 30 t$. Adding $t' \neq 0$ breaks the particle-hole symmetry⁷ and even at half-filling LSR could become questionable⁸. Nevertheless, choosing $t' = -0.2 t$ we also do not find any evident breakdown following scenario IV of LSR at or away from half-filling.

Several regimes with the violation of LSR (e.g., $N = 6$ electrons on $N_0 = 14$ sites for $U > 6 t$) can be mainly attributed to level crossing of $|0_{N+1}\rangle$ or $|0_{N-1}\rangle$ (scenario II). Also for half filled $N_0 = 12$ system, where the degeneracy of the g.s. is removed by choosing small θ we find $N' = 10$ particles in LV instead of $N = 12$ above $U_c(\theta) < 4 t$, which seems to represent the violation of type III. In this case $N-1, N, N+1$ g.s. preserve symmetry of NIE, however $|0_N\rangle$ has nonzero \mathbf{K} .

2D systems: $t' = 0$ Hubbard model we investigate on square lattices with $N_0 = 8, 10, 16$ sites. As a general rule (with some exceptions elaborated below), we find that the LSR remains satisfied for all considered $N \leq N_0$ and moreover the topology of the LV remains that of NIE. In particular, we find no violation for half filling $N = N_0$, consistent with particle-hole symmetry^{7,8}.

Away from $N = N_0$ one exceptional deviating case is $N = 6$ on $N_0 = 8$ sites where at larger $U > U_c$ the LSR yields $N' = 8, 10, 14 > N$. Again, in this case NIE g.s. is degenerate and U_c is θ dependent and also for $U < U_c$ level crossing in $N-1, N+1$ g.s. is observed. All this indicates on combination of scenarios II and III.

In our study of 2D t - t' - U model we use systems with $N_0 = 8, 10, 16$ sites and $t' = -0.3t$ as frequently invoked for superconducting cuprates. We find no clear (type IV) violation of LSR in range $0 < U < 40 t$ and different fillings $N \leq N_0$. However, some deviations from LSR connected with level crossing scenario II are found at and away from half-filling even for closed shell configurations. E.g., $N = 14$ electrons on $N_0 = 16$ sites at $U > 30 t$ result in $N' = 20$ within the LV. Although $|0_N\rangle$ g.s. remains throughout $S = 0$ and $K = 0$, in this case $N+1$ g.s. changes from NIE value $S = 1/2$ to higher spin $S > 1/2$ at $U > 30 t$.

B. t - J model

Another prototype model for strongly correlated electrons is the t - J model,

$$H = - \sum_{i,j,s} t_{ij} \tilde{c}_{js}^\dagger \tilde{c}_{is} + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j), \quad (9)$$

where $\tilde{c}_{is} = c_{is}(1 - n_{i,-s})$ are projected operators not allowing for a double occupancy at each site, and \mathbf{S}_i are the local spin operators coupled with the exchange interaction J . The t - J model can be considered as the truncated Hubbard model at large $U \gg t$ and $J = 4t^2/U$. It is assumed that low-energy physics of both models is similar, therefore it is of interest whether they behave similarly with respect to LSR. We note that formally t - J model can be also written as the tight-binding model, Eq. (6), with the interaction

$$H_{int} = J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) + \tilde{U} \sum_i n_{i\uparrow} n_{i\downarrow}, \quad (10)$$

whereby we let $\tilde{U} \rightarrow \infty$ to get a projected model, Eq. (9). Such a formulation helps to formally connect the t - J model with finite $J > 0$ continuously with NIE. Although the model is not perturbative in any limit, one can first consider the $J > 0$ term as the perturbation and then performing the limit $\tilde{U} \rightarrow \infty$. In this way it makes sense to compare the LSR within the t - J model directly to NIE with the dispersion, Eq.(8).

1D systems: Within the t - J model it makes sense to discuss cases $N < N_0$, i.e. representing the Mott insulator doped with $N_h = N_0 - N$ holes. The model on a chain at $J \rightarrow 0$ behaves as the $U = \infty$ Hubbard model obeying LSR as confirmed in our numerical calculations. Finite $J > 0$ is nontrivial and some breakdowns of LSR are found at low doping $N_h \ll N_0$. For $N_0 = 16$ and $N_h = 2$, $N = 14$ representing a closed shell configuration we find at $J > 0.7 t$ $N' = 24$. Since the $|0_N\rangle$ has $S = 0$, $K = 0$ as well as $N - 1, N + 1$ g.s. have $S = 1/2$, this case represents a clean violation of LSR of type IV.

2D systems: Here, one should also monitor the concavity of E_N^0 which can be violated at low doping, but not for even N considered in our study. In general, results and conclusions for the LV within the t - J model are quite similar to the corresponding large- U Hubbard model. Essential differences appear at higher parameter $J > 0.1 t$ and at $N_h \ll N_0$. We investigate further on lattices with $N_0 = 16, 18, 20$ sites and $N_h < 6$.

The most clear counterexample of LSR following scenario IV is the closed-shell system with $N_h = 2$ on $N_0 = 20$ sites. The corresponding LV for NIE is shown Fig. 1a, together with wave-vectors appearing on a $N_0 = 20$ lattice. On the other hand, we find that for $0.1 t < J < 0.3 t$ the LV contains $N' = 22$ particles, as shown in Fig. 1b. Here, $|0_N\rangle$ has $S = 0$, $K = 0$, and no level crossing in $N - 1, N + 1$ g.s. with $S = 1/2$ is detected in this regime, excluding scenario II. For larger J/t even more drastic violations are found. For $J = 0.4 t$, $J = 0.5 t$ we find $N' = 24$ and $N' = 32$, respectively, as represented in Figs. 1c,d. It is expected that introduction of t' should even increase the deviation from the LSR¹⁹.

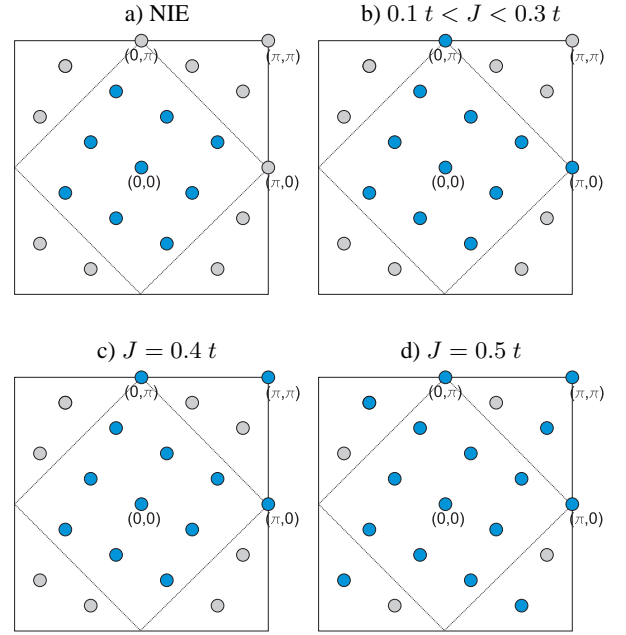


Figure 1: Luttinger volume for closed shell configuration of $N = 18$ electrons on $N_0 = 20$ square lattice for: a) noninteracting electrons, b) t - J model with $0.1 t < J < 0.3 t$, c) $J = 0.4 t$, and d) $J = 0.5 t$. Gray (blue) circles represents (nonequivalent) wave vectors \mathbf{k} outside (inside) the Luttinger volume.

Results for the violation of the LSR, presented in Figs. 1b-d are not unexpected and qualitatively consistent with other (indirect) indications of the violation of the LSR within the t - J model, obtained via the high- T expansion of momentum distribution $n_{\mathbf{k}}$ ¹³ or a straightforward interpretation of spectral functions $A(\mathbf{k}, \omega)$, calculated on small systems¹⁹. Namely, in the range of parameters for cuprates $J \sim 0.3 t$ results within the t - J model show too large LV, $N' > N$, whereby the violation is modest^{13,19}, the conclusion is consistent with experimental finding for hole doped cuprates^{11,12}.

In a doped system larger J plausibly enhances the antiferromagnetic order (remaining always finite-range in small systems) which can lead to the formation of hole pockets, which should show up in small Fermi surface and corresponding LV forming around $\mathbf{k} = (\pi/2, \pi/2)$. Indeed such tendency is evident in Figs. 1c,d where at largest $J = 0.5 t$ only the point closest to $\mathbf{k} = (\pi/2, \pi/2)$ remains outside the LV.

IV. CONCLUSIONS

In conclusion, we have shown that the investigation of finite-size systems can provide a non-trivial test of the LSR and its validity for strongly correlated systems. Several properties of finite systems, in particular the degeneracy of g.s., can complicate the interpretation of results. Nevertheless, the cases with the major violation of the LSR should be easily detectable also in small systems. In this study we examine only prototype Hubbard and t - J models, still the application of the method to other models of correlated electrons is straightfor-

ward.

It should be pointed out that in all evident (type IV) cases the breakdown of the LSR appears by a continuous variation of GF zero $\omega_0 \sim 0$ across the chemical potential, which in a finite system is connected with a divergence of the corresponding $\Sigma(\mathbf{k}, \omega \sim 0) \rightarrow \infty$. Such a transition remains an evident possibility in an insulator even in the limit $N \rightarrow \infty$ ⁸. On the other hand, in a metal (where in a normal FL the Fermi surface is determined by poles of GF) such a scenario would represent a major modification of the LV concept and therefore the limit $N \rightarrow \infty$ allowing for several nontrivial scenarios should be considered with care.

For the Hubbard model on 1D chain and 2D square lattice, both for $t' = 0$ and $t' \neq 0$, we do not find a clearcut violation (type IV) of the LSR. While this is rather a test of our approach for 1D systems where exact results^{16,17} indicate that for $N \rightarrow \infty$ the LV remains that of the NIE with the singularity at $k_F = \pi N/2N_0$ away from half filling. For 2D system the validity of the LSR would represent the confirmation of the fact that the g.s. even at large $U > 0$ is adiabatically connected to that of NIE, as far it represents the paramagnetic state. Several examples of type II, III deviations still point on a cautious interpretation, since systems studied numerically are small. In particular for 2D lattice, large separation of discrete levels for NIE in considered systems indicate that it would be hard to observe small deviations from LSR. Another possibility in $D > 1$ is a change of the Luttinger-surface form while preserving LV, allowed and expected for $U > 0$. In our approach the latter would show up with few zeroes of GF entering and leaving the LV. Since nonequivalent zeros can hardly cross μ

at the same parameter in a finite system, this would lead to intermediate violation of LSR. Again, our systems seem to be too small to observe this phenomenon.

Within the t - J model we observe the violation of the LSR, particularly evident for the 2D systems corresponding to low doping of the Mott insulator as relevant to superconducting cuprates. This can be interpreted the g.s. of the model not being adiabatically connected to the g.s. of NIE, although formally the latter would be allowed by performing the perturbation in $J > 0$ and letting $\tilde{U} \rightarrow \infty$ in Eq. (10). It is not surprising that the latter extrapolation can break the connection to NIE. In our study we follow numerically only $J > 0.1 t$. On the other hand, it is clear that $J \rightarrow 0$ case should be equivalent to the Hubbard model with $U \rightarrow \infty$. In the latter regime, the Hubbard model (or t - J model with $J \rightarrow 0$) close to half-filling can show instability towards partly or fully spin polarized states $S \gg 0$ as well as more pronounced finite-size effects, which are only partly studied in the present work.

Our results are consistent with other theoretical indications for the deviations from LSR^{13,19} as well as experiments on cuprates^{11,12}. Still the speculations on the origin and the extrapolation to a macroscopic system are delicate. While the deviation from the LSR could indicate a general breakdown of the Fermi liquid concept or be a sign of inherent insufficiency of the model, the deviations could as well disappear in the limit $N \rightarrow \infty$ in a metallic system while persisting (or not) e.g. in a Mott insulating state.

One of the authors (P.P.) thanks M. Potthof for useful suggestions regarding the LSR.

¹ J. M. Luttinger and J. C. Ward, Phys. Rev. **118**, 1417 (1960).

² J. M. Luttinger, Phys. Rev. **119**, 1153 (1960).

³ A. A. Abrikosov, L. P. Gorkov, I. E. Dzialoshinski, *Methods of quantum field theory in statistical physics*, Dover Publ., NY, (1975).

⁴ K. B. Blagoev and K. S. Bedell, Phys. Rev. Lett. **79**, 1106 (1997).

⁵ M. Yamanaka, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. **79**, 1110 (1997).

⁶ I. Dzyaloshinskii, Phys. Rev. B **68**, 085113 (2003).

⁷ T. Stanescu, P. W. Phillips, and T.-P. Choy, cond-mat/0602280.

⁸ A. Rosch, cond-mat/0602656.

⁹ R. M. Konik, T. M. Rice, and A. M. Tsvelik, Phys. Rev. Lett. **96**, 086407 (2006).

¹⁰ B. L. Altshuler, A. V. Chubukov, A. Dashevskii, A. M. Finkel'stein, and D. K. Morr, Europhys. Lett. **41**, 401 (1998).

¹¹ K. M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle,

W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005).

¹² T. Yoshida, X. J. Zhou, K. Tanaka, W. L. Yang, Z. Hussain, Z.-X. Shen, A. Fujimori, S. Komiyama, Y. Ando, H. Eisaki, T. Kakeshita, S. Uchida, cond-mat/0510608.

¹³ W. O. Putikka, M. U. Luchini, and R. R. P. Singh, Phys. Rev. Lett. **81**, 2966 (1998).

¹⁴ Th. A. Maier, Th. Pruschke, and M. Jarrell, Phys. Rev. B **66**, 075102 (2002).

¹⁵ A. Praz, J. Feldman, H. Knoerr, and E. Trubowitz, Europhys. Lett. **72**, 49 (2005).

¹⁶ E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. **20**, 1445 (1968).

¹⁷ H. Frahm and V. E. Korepin, Phys. Rev. B **42**, 10553 (1990).

¹⁸ E. Dagotto, Rev. Mod. Phys. **66**, 763 (1994).

¹⁹ M. Zemljic and P. Prelovsek, cond-mat/0608310.